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Elasticity Constants of Single Crystals of Nickel-Copper Alloys*

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Synopsis

Single crystals of Ni-Cu alloys were prepared by the Bridgman method of slow solidification, and Young's moduli were measured at room temperature by a new microscopic method. From the measured values, Young's moduli for the principal orientations were derived. Young's modulus in [100] direction decreases at first with the addition of copper to nickel, reaches the minimum at about 10, the maximum at about 25 per cent copper, and then decreases with further increasing content of copper. The behavior of Young's modulus in [110] direction is similar to that of E_{100}, while in [111] direction it decreases monotonously. The three principal elastic coefficients S_{11}, S_{12} and S_{44} were determined by using the data of compressibility for polycrystals of Ni, Cu and a Ni-Cu alloy containing 45 per cent of copper. Futhermore, the principal rigidity moduli were derived.

I. Introduction

It is very significant to measure the elastic constants of single crystals of the pure ferromagnetic metals and alloys for the fundamental researches on their physical properties. Therefore, many studies have already been performed on Fe,[1] Co,[2] Ni[3] single crystals. But, there are very few about single crystals of alloys, especially for single crystals of Ni-Cu alloys.

Such being the case, in the present study Young's modulus was measured for single crystals of Ni-Cu alloys by the microscopic method[4] by the late Prof. Honda and one of the present authors.

II. Specimens, apparatus for experiment and method of measurement

Specimens of single crystals were prepared by the Bridgman method of slow solidification. First, the alloys were melted in a high frequency induction furnace, then forged and cold-drawn to wire, 3 mm in diameter. The chemical analyses of metals used to make alloys are shown in Table 1. Single crystals of 13 kinds of alloys were prepared, copper contents of which are shown in Table 2. The chemical analyses of alloys are shown also in Table 2, in which the copper contents of the

* The 899th report of the Research Institute for Iron, Steel and Other Metals. The original of this report as written in Japanese was previously published in Nippon Kinzoku Gakkai-shi (J. Japan Inst. Metals), **19** (1955), 99.

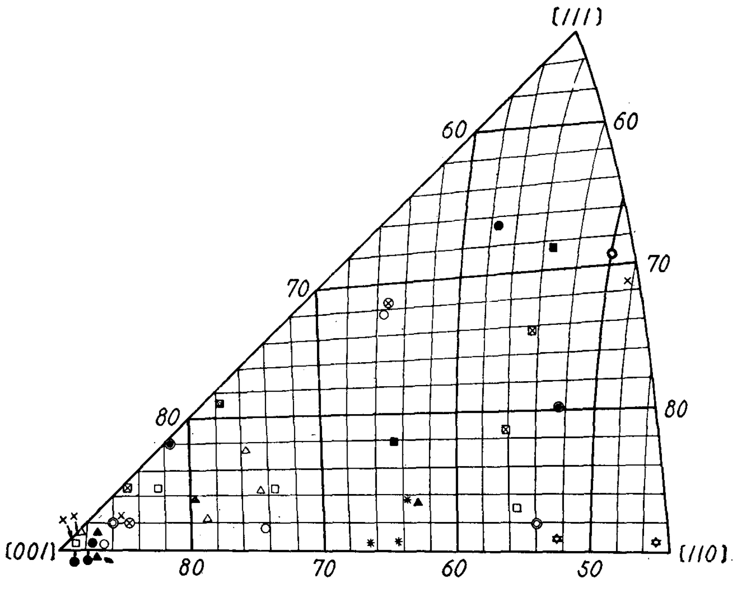
(1) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **7** (1943), 346.
(2) K. Honda and Y. Shirakawa, Nippon Kinzoku Gakkai-shi, **13** (1949), 23; Sci. Rep. RITU., **A1** (1949), 9.
(3) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **6** (1942), 331; Sci. Rep. RITU., **A3** (1951), 308.
(4) K. Honda and Y. Shirakawa, Nippon Kinzoku Gakkai-shi, **1** (1937), 217.

Table 1. Chemical analyses of metals used.

Metals	C (%)	Si (%)	Mn (%)	Fe (%)	Al (%)	Co (%)	As (%)
Electrolytic nickel	0.056	0.017	0.004	0.054	none	0.022	none
Electrolytic copper	0.029	0.005	0.0008	0.0051	trace	none	trace

Table 2. Chemical analyses, elastic constants S_{ik}, and elastic moduli E of principal crystal directions for Ni-Cu alloys.

Specimen	Cu	Young's moduli (10^{12} dyne/cm^2)			Elastic coefficients (10^{-12}cm^2/dyne)		
No.	(%)	$E_{[100]}$	$E_{[110]}$	$E_{[111]}$	S_{11}	S_{12}	S_{44}
Ni	—	1.07	1.79	2.30	0.939	−0.374	1.108
1	5.10	0.98	1.70	2.25	1.026	−0.417	1.142
2	9.82	0.95	1.67	2.23	1.053	−0.429	1.154
3	15.48	1.00	1.69	2.19	1.000	−0.401	1.174
4	20.38	1.06	1.71	2.14	0.943	−0.371	1.198
5	25.65	1.07	1.69	2.10	0.940	−0.368	1.228
6	30.42	1.04	1.64	2.02	0.965	−0.379	1.278
7	44.15	0.94	1.46	1.81	1.063	−0.424	1.460
8	49.59	0.91	1.41	1.73	1.102	−0.442	1.520
9	60.69	0.89	1.35	1.63	1.130	−0.453	1.634
10	70.20	0.84	1.26	1.50	1.190	−0.481	1.762
11	80.47	0.77	1.17	1.42	1.294	−0.530	1.892
12	90.42	0.75	1.14	1.37	1.330	−0.546	1.960



No.	Cu (%)		No.	Cu (%)		No.	Cu (%)	
Ni		○	5	25.65	■	10	70.20	⊠
1	5.10	□	6	30.42	▲	11	80.47	*
2	9.82	△	7	44.15	◎	12	90.42	✡
3	15.48	×	8	49.59	◉			
4	20.38	●	9	60.69	⊗			

Fig. 1. Distribution of single crystals of Ni-Cu alloys used on the stereographic net.

specimens 8~12 were calculated from the amounts of nickel analyzed. Orientations of specimens were determined by the light-figure method,(5) and are shown stereographically in Fig. 1. They were annealed at respective recrystallization temperatures for 3 hours in hydrogen atmosphere and then cooled slowly, and were further annealed at 700°C for one hour in high vacuum before every experiment Young's moduli were measured at room temperathre by the abobe-mentioned microscopic method.(2)(4)

Denote the weight by W, the distance between the knife edges or the length of span by l, the width and the thickness of specimen by b and d, respectively, and the deflection of specimen due to the weight W by h, then Young's modulus E will be given by the formula

$$E = \frac{Wl^3}{4bd^3h}. \tag{1}$$

In the present experiment,

$$l = 3.002 \text{ or } 4.050 \text{ cm},$$
$$b = 0.1270\sim0.2315 \text{ cm},$$
$$d = 0.107\sim0.1930 \text{ cm}.$$

The weight on the dish, 5.8 g in weight, was 50 or 100 g. The deflection h of the center by the weight is measured by the microscope with the magnification of 1200 times.

For example, the orientations, dimensions, and deflections for 2 specimens containing 5.10 and 9.81 per cent copper are shown in Table 3. The maximum stress applied was $0.88\sim2.04 \times 10^8$ dyne/cm^2.

Table 3. Orientations, dimensions, deflections by load h and Young's moduli E of specimens of Ni-Cu alloys. α and β denote angles between the rod axis of specimen crystal and any two tetragonal axes; Γ orientation function, l the length, b the width, d the thickness.

Specimen No.	α degree	β degree	3Γ	l cm	b cm	d cm	h (10^{-3}cm)	E (10^{12}dyne/cm^2)
5.10%Cu-1	89.7	89.3	0.000	3.002	0.1635	0.1569	1.09	0.98
5.10%Cu-2	85.4	73.6	0.238	3.002	0.1980	0.1510	0.86	1.13
5.10%Cu-3	85.3	82.4	0.071	3.002	0.1510	0.1230	2.34	1.01
5.10%Cu-4	86.9	55.3	0.663	4.050	0.2150	0.160	1.18	1.56
9.82%Cu-1	82.3	75.9	0.218	3.002	0.196	0.129	1.46	1.08
9.82%Cu-2	85.4	74.3	0.222	4.050	0.199	0.163	1.73	1.09
9.82%Cu-3	88.6	88.3	0.000	3.002	0.203	0.159	0.85	0.95
9.82%Cu-4	87.5	78.9	0.113	3.002	0.201	0.162	0.76	1.02

(5) M. Yamamoto and J. Watanabe, Nippon Kinzoku Gakkai-shi, **18** (1954), 595.

III. Experimental results and the consideration

1. Young's moduli

Young's moduli were measured on the alloy crystals of the same composition with different orientations, is shown in Table 3. The relation between the reciprocal of Young's moduli $1/E$ and the orientation function 3Γ ($\Gamma = \gamma_1^2\gamma_2^2 + \gamma_2^2\gamma_3^2 + \gamma_3^2\gamma_1^2$, γ_i being direction cosines of the specimen axis referred to the crystal axes,) is shown in Fig. 2. In general, Young's modulus E of a cubic crystal is expressed as follows:

$$1/E = S_{11} - 2(S_{11} - S_{12} - 1/2S_{44})\Gamma, \tag{2}$$

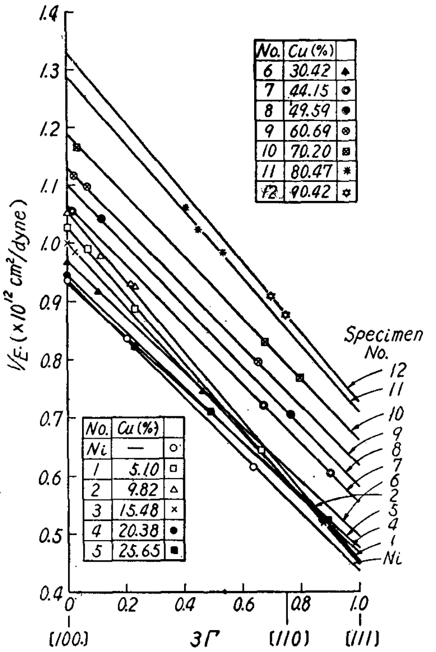


Fig. 2. Relation between $1/E$ and orientation function $3\Gamma = 3(\gamma_1^2\gamma_2^2 + \gamma_2^2\gamma_3 + \gamma_3^2\gamma_1^2)$ for the single crystals of Ni-Cu alloys.

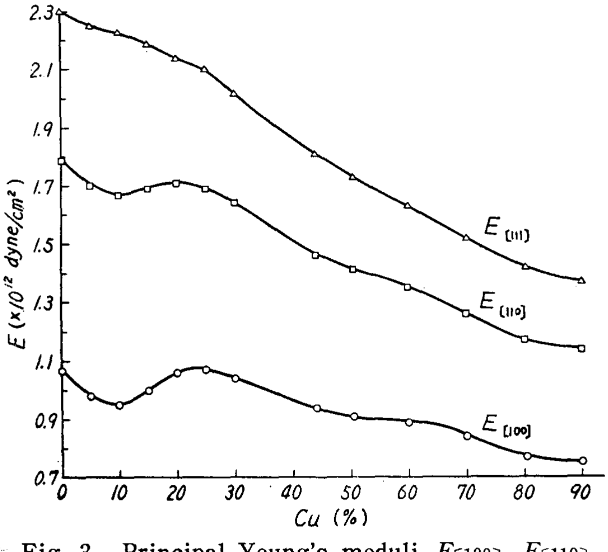


Fig. 3. Principal Young's moduli $E_{[100]}$, $E_{[110]}$ and $E_{[111]}$ vs. concentration curves for Ni-Cu alloys.

where S_{ik}'s are Voigt's notation of elastic parameters. Young's moduli in the three principal axes [100], [110] and [111] calculated by the above formula are shown in Table 2. Fig. 3 shows the principal Young's moduli E_{100}, E_{110} and E_{111} against the concentration for Ni-Cu alloys.

Young's moduli for the [100], [110] and [111] directions of Ni were 1.07, 1.79 and 2.30×10^{12} dyne/cm² respectively. These are a little smaller than those by Honda and one of the present authors(2), and the anisotropy a little larger. From Fig. 3, it will be seen that Young's moduli E_{100} and E_{110} vs. concentration curves for Ni-Cu alloys are very similar to one another, that is, Young's modulus of Ni decreases with the addition of copper, and shows a minimum at 10 per cent of copper, and then passing through a maximum point at 20~25 per cent of copper, decreases very simply except a slight curvature at 60~70 per cent copper. However, the curve of Young's moduli E_{111} is very different from those of E_{100} and E_{110}, and Young's moduli E_{111} of Ni (2.3×10^{12} dyne/cm²) simply decreases with the increase of copper content, showing neither minimum nor maximum.

2. Elastic constants and rigidity moduli

Elastic parameters S_{ik}'s cannot be determined from Eq. (2), because it shows only S_{11} and $(S_{11}-S_{12}-1/2S_{44})$. Therefore, S_{12} and S_{44} were determined from the compressibility of polycrystalline alloy. The compressibility k will be the same for a single crystal as for a polycrystal, if the effect of grain boundaries is negligible, as verified by many exyeriments(3).

The compressibility for Ni-Cu alloy has not yet been measured systematically, being reported only for Ni(6), Cu(7) and constantan (45 per cent copper)(6), with the values respectively of 0.570, 0.732 and 0.645×10^{-12} cm^2/dyne. Unexpectedly, these values lie on a straight line against copper content. Of course, it will be considered that the compressibility for nickel side does not necessarily lie on a straight line because of their ferromagnetism, but it was intended to determine the compressibilities approximately by interpolation on account of low sensitivity of the present experiment.

The compressibility k is gixen by the following relation:

$$k = 3(S_{11} + 2S_{12}). \tag{3}$$

Three principal parameters S_{ik}'s were determined by Eqs. (2) and (3), and the result is shown in Table 2. As seen in the table, S_{12} was negative. When only the absolute values are taken, the relations between the principal elastic parameters S_{11}, S_{12} and S_{44} and copper content are almost similar to those between the reciprocal of Young's moduli E_{100}, E_{110} and E_{111} and copper content, respectively. As the principal elastic parameters S_{ik}'s were determined, the rigidity modulus G could be determined by the following equation:

$$1/G = S_{44} + 4(S_{11} - S_{12} - 1/2\,S_{44})\Gamma. \tag{4}$$

The relations between the rigidity moduli in three principal orientations, G_{100}, G_{110} and G_{111}, and copper content are shown in Fig. 4. The tendency of curves is similar to that of Young's modulus. Further, from three principal elastic parameters S_{ik}'s, three principal elastic constants C_{ik}'s were computed, resulting in the invalidity of Cauchy's relation $C_{12} = C_{44}$ as anticipated.

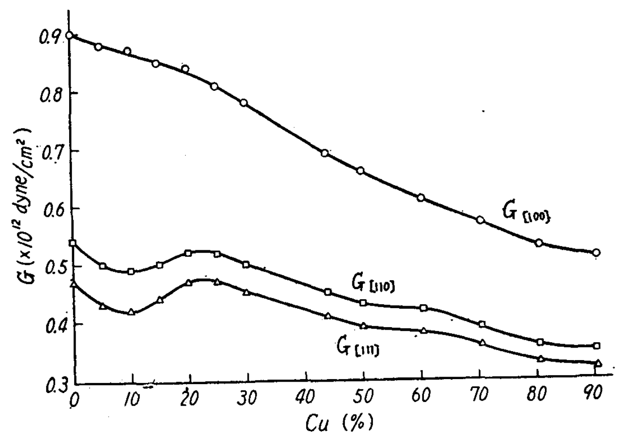


Fig. 4. Principal rigidity moduli $G_{[100]}$, $G_{[110]}$ and $G_{[111]}$ vs. concentration curves for Ni-Cu alloys.

The degree of elastic anisotropy, that is, $2(S_{11} - S_{12})/S_{44} = 2C_{44}(C_{11} - C_{12})$ vs. concentration curve is shown in Fig. 5. The degree of elastic anisotropy increases with added copper to nickel until the

(6) U. Yoshida and H. Takei, *Experimental Physics, 5th Ed., Tokyo* (1941), 269.
(7) R. Kimura, Sci. Rep. Tôhoku Univ., **22** (1933), 553.

maximum point at 10 per cent copper and then decreases first suddenly and then comparatively slowly till above 30 per cent copper. In this case, a slight curvature is seen at 60～70 per cent copper. The values of Γ are expressed as a function of the angle θ between the cubic axis in the three principal planes (100), (110) and (111). As an instance, polar diagrams expressing the distribution of Young's moduli E and rigidity modulus G in the two principal planes (100) and (110) for the alloy containing 9.82 per cent copper are shown in Fig. 6. In addition, Γ in the plane (111) is 1/4, and so E and G on the plane (111) are both isotropic.

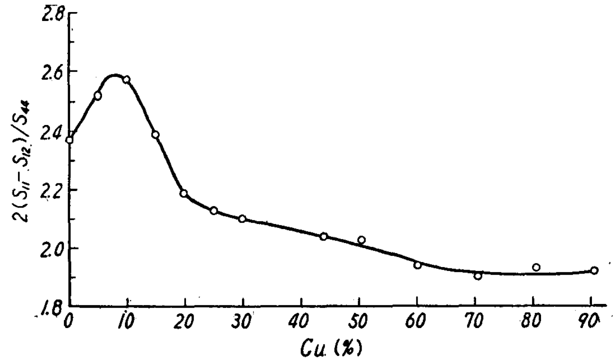


Fig. 5. Degree of elastic anisotropy, $2(S_{11}-S_{12})/S_{44} = 2C_{44}/(C_{11}-C_{12})$ vs. concentration curve for Ni-Cu Alloys.

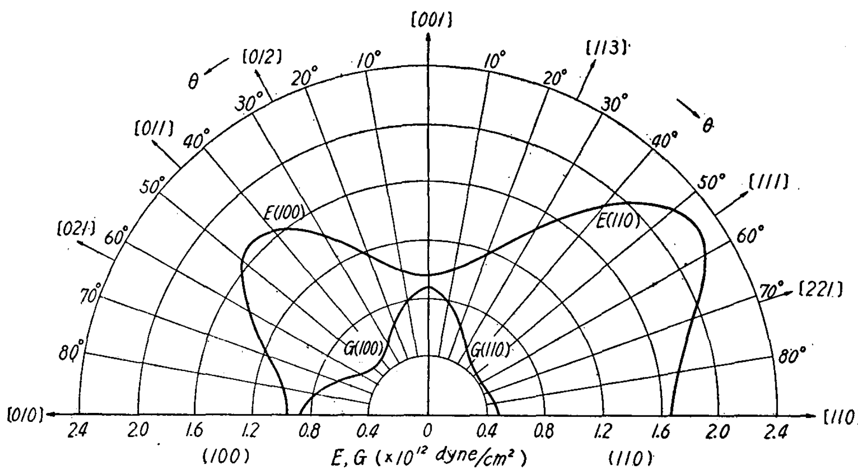


Fig. 6. Polar diagram constructed from E and G for single crystals of 9.82 Cu alloy in the planes (100) and (110).

3. Young's moduli for quasi-isotropic polycrystals

By averaging the elastic constants of single crystals, Young's and rigidity moduli for a quasi-isotropic polycrystal can be calculated. Young's modulus of polycrystals of Ni-Cu alloys have been measured by many workers[(8)], but their results in ferromagnetic range are very different from one another, and even the tendencies of E vs. copper concentration curves are also different with different workers. Hence, E and G of a polycrystal should not be calculated by using the elastic constants S_{ik} and C_{ik} computed from the results of single crystals.

In the present study, it was considered that the crystal grain size would be one of important factors that produce the above-mentioned inagreement; the results will be reported in detail in the next paper.

(8) S. Umekawa, Nippon Kinzoku Gakkai-shi, **18** (1954), 387.

Summary

(1) 44 single crystals of 13 kinds of Ni–Cu alloys including Ni were prepared by the Bridgman method, and Young's moduli were measured at room temperature by the microscopic method.

(2) From the relation between the reciprocal of the modulus $1/E$ and orientation functions $\Gamma = \gamma_i^2\gamma_k^2$ of specimens, Young's moduli in the three principal crystallographic directions E_{100}, E_{110} and E_{111} were obtained. E_{100} and E_{110} vs. copper concentration curves were similar to each other and showed a minimum and maximum at 10 and 20 per cent of copper, respectively, and a slight curvature at 60~70 per cent of copper. The curve for E_{111}, however, was smooth, decreasing from the value of Ni, 2.3×10^{12} dyne/cm^2, to that of Cu.

(3) The principal elastic parameters S_{ik}'s were calculated from the observed values of Young's moduli, and the compressibility of polycrystal was determined by interpolation from the values of 3 alloys of Ni–Cu series. The value of S_{12} was negative, but taking only the absolute value into consideration, the relationships between S_{11}, S_{12}, S_{44} and copper content were reverse to those for E_{100}, E_{110} and E_{111}, respectively.

(4) The rigidity moduli of the principal directions, G_{100}, G_{110} and G_{111} were computed from the principal elastic parameters S_{ik}'s. G_{100} vs. copper concentration curve was simply descending, decreasing from the value of Ni, 0.9×10^{12} dyne/cm^2, but those of G_{110} and G_{111} were similar to E_{110} and E_{100}, respectively.

(5) The degree of elastic anisotropy of Ni increased with the increase in copper content until a maximum point at 10 per cent of copper, and then decreases suddenly after 10 per cent of copper, and comparatively smoothly above 30 per cent of copper.

In conclusion, the present writers wish to express their hearty thanks to Mr. D. Amemiya for his strenuous exertion made in the preparation of single crystals.